Investigating Fraction Understanding Across Music and Mathematics

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Abstract

From dividing a pizza among friends to adjusting time schedules, fractions silently weave into our daily routines, highlighting the necessity of grasping their nuances. However, when dealing with fractions, people often intuitively apply their knowledge of whole numbers, causing them to make errors in judgement. This bias towards whole number logic leads to incorrect answers, or slower response times when comparing fractions. Due to their exposure to fractions through musical notation, musicians offer a valuable perspective to fraction understanding outside of mathematical contexts. The present study investigated this whole number bias in undergraduate students with and without formal musical training to determine if their fluent use of fractions would transfer to the mathematical domain. Participants were asked to compare fractions; half of the fraction comparisons had answers that were congruent with the intuitive strategy, while the other half were incongruent with the intuitive reasoning. In this study, 'congruent' comparisons refer to fraction pairs where the intuitive strategy aligns with the correct answer, meaning that the fraction with the numerically larger value also contains larger numerators and denominators. Conversely, 'incongruent' comparisons denote fraction pairs where the intuitive strategy leads to an incorrect judgment, as the numerators and denominators of a fraction may be larger but may have the smaller magnitude. We found that only musicians specifically display better performance for fractions used in the context of music, compared to other fraction pairs. These findings could have implications for mathematical education strategies and serve as a foundation for future studies to examine fraction usage outside of a mathematical context.

Investigating Fraction Understanding Across Music and Mathematics

From understanding player statistics on sports teams to adjusting ingredient quantities in recipes, fractions play an essential role in our lives. With their prevalence, it is remarkable that fractions still pose challenges for many individuals. Humans across diverse demographics grapple with the "whole number bias", where they mistakenly apply whole-number logic to fractions (Ni & Zhou, 2015). This phenomenon occurs in tasks involving fraction processing, such as comparisons. When asked to compare and identify the larger of two, two approaches are commonly used. Individuals employing a holistic approach perceive fraction magnitudes as the absolute ratio magnitude (Leibovich et al., 2017). In contrast, those using the componential approach analyse numerators and denominators separately for comparison (Leibovich et al., 2017).

Componential approaches tend to be a faster method of evaluation and are often the intuitively preferred method of fraction comparison (Vamvakoussi et al., 2012). However, the latter strategy does not always lead to desirable outcomes. If the components of the greater fraction are greater than the components of the lesser fraction (e.g., 8/9 > 5/7), then the comparison is said to be congruent, and the componential approach will be successful. Otherwise, if the components of the greater fraction are lesser than those of the lesser fraction (e.g., 3/7 vs 2/3), then the comparison is considered incongruent, and the componential approach fails. The componential approach to fraction comparison will therefore only work with congruent comparisons.

The whole number bias is present in a wide variety of populations. At the elementary and high school education level, a common mistake that students make is using the componential approach for incongruent comparisons (Meert et al., 2010b; Vamvakoussi & Vosniadou, 2004).

This phenomenon occurs in western, indigenous, and otherwise non-western populations (Braithwaite & Siegler, 2018; Alonzo-Dìaz et al., 2019; Bailey et al., 2015; Lai and Wong, 2017). The whole number bias does not disappear even after education, although it may manifest itself differently. Adults typically display fewer inaccuracies when presented with fraction comparison tasks (Vamvakoussi et al., 2012). The whole number bias is instead reflected in their response time; undergraduate students have been shown to take longer to respond to incongruent comparison types, compared to congruent comparisons (Vamvakoussi et al., 2012; DeWolf & Vosniadou, 2015). This phenomenon is present regardless of mathematical expertise; studies demonstrate that expert mathematicians lean towards the intuitive process of fraction comparison when dealing with fractions with common components (i.e., same numerator or denominator), demonstrating the whole number bias for the incongruent comparisons (Obersteiner et al., 2013).

Dual-processing theories might account for the difference in response times. Under these theories, there are two posited streams of cognitive processing: intuitive and analytical processing (Gillard et al., 2009). Within fraction comparison, the componential approach falls under intuitive processing, while the holistic approach falls under analytical processing (Meert et al., 2010b). These two processes are distinct, with the intuitive process preceding the analytical process. Therefore, when an adult responds to incongruent comparisons, they first lean toward the componential comparison and then suppress the process in favour of the more analytical holistic approach (Meert et al., 2010a, 2010b). The whole number bias is then manifested in the adult population due to this extra step of suppressing the intuitive process.

Fractions are utilized beyond mathematics; for instance, in Western musical notation, musical notes are named using fractions, representing their duration relative to a whole note (e.g., a quarter note is ¼ of the duration of a whole note; The Royal Conservatory of Music, 2016). Additionally, when reading sheet music, musicians often think of other notes proportionally to quarter notes (e.g., an eighth note is half of a quarter note in duration, and a half note is twice as long as a quarter note; Palmer & Krumhansl, 1990). Remarkably, musicians accurately discriminate between note durations, correctly identifying the shorter of two note values despite the comparison being incongruent, like in the case of a quarter note and a half note (Kamrlová & Varhaníková, 2011). Furthermore, musicians practically use fraction equivalence frequently, as they often subdivide note lengths into smaller but more numerous proportions to play rhythms with greater precision (Drake, 1993). There are even circumstances where musicians will use fraction arithmetic, as is the case with dotted or tied notes, where note durations of different lengths are added together; musicians will play rhythms involving these accurately at various tempos, illustrating their understanding of this concept (The Royal Conservatory of Music, 2016).

Musicians portray fraction fluency in their music domains through musical notation, making them an interesting population in which to study fraction understanding. However, there is currently an absence of literature exploring the presence of the whole number bias in musicians. The research question of the present study asks whether musicians experience the whole number bias to the same degree as non-musicians. The study's primary goal was to determine whether there were any differences between musicians and non-musicians in processing fraction comparisons within a mathematical context. Anticipating a potential transfer of their fluency with fractions from music to mathematics, we expected that musicians would be less susceptible to the whole number bias compared to non-musicians.

In this study, musicians and non-musicians completed a fraction comparison task. In accordance with previous literature (Obersteiner et al., 2016; Vamvakoussi et al., 2012), we expected to see high accuracy across both participant groups. We instead predicted that differences would be reflected in participants' response times. We hypothesized that if musicians are impacted to a lesser degree by the whole number bias, then musicians would display shorter response times compared to non-musicians specifically in incongruent comparisons. A secondary goal was to identify if any differences were specific to fractions that are often seen within a musical context. If musicians have shorter response times due to a transfer of their skills from a musical to a mathematic domain, then we hypothesize that differences in response times for musicians and non-musicians will be even greater for incongruent comparisons featuring fractions more familiar in a musical context. Such a finding would support the notion that it is because musicians are taught fractions in a different context that allows them to have shorter response times. This transfer of knowledge would not have to be limited to musicians but could also include other professions that work with fractions frequently, such as bakers or construction workers. If transfer of fraction fluency between domains is indeed demonstrated, it can inform educators to develop strategies that approach fraction learning holistically, beyond numerical representations. By integrating non-numerical perspectives, such as musical contexts, educators may be able to encourage students to perceive fractions more holistically and intuitively.

Method

Participants

Sixty-five undergraduate students (36 female, aged 18 - 23 years, M = 18.7, SD = 1.2) at the University of Western Ontario were recruited through the psychology research participation pool on SONA Systems. Students were separated into two groups post hoc based on their responses to a demographic questionnaire. Trained musicians (n = 22) had at least five years of formal music training, could fluently read Western rhythmic notation, and had regularly engaged in musical instrument practice within the last three years (operationalized as at least weekly). Non-musicians (n = 43), served as the control and were characterized by their lack of music training experience and unfamiliarity with Western music notation. All participants provided informed consent and were compensated 1.0 study credits for their introductory psychology course. This study was approved by the Non-Medical Research Ethics Board.

Materials

Participants participated remotely, using personal computers with keyboards. All experimental tasks were built through PsychoPy-2023.2.3 (Pierce et al., 2019), developed by Open Science Tools Ltd., and were hosted online using Pavlovia (Pierce et al., 2019). The CSV files were created using Python. All tasks were embedded in a Qualtrics questionnaire. Analyses were performed using R.

Procedure

First, participants completed a fraction comparison task in which they had to compare fractions to identify the larger of the two magnitudes. Next, to ensure that participants' math fluency was not a confound, participants completed an arithmetic task, in which they evaluated as many arithmetic problems as they could in three minutes. The last task that participants did was a music note comparison, in which they identified the larger of two music note lengths.

Fraction Comparison Task

In the fraction comparison task, participants were shown a fixation cross, followed by two fractions, and were asked to select the greater of the two fractions. Participants tapped the 'F' key or 'J' key to select the fraction on the left or right, respectively. In the practice trials, participants compared whole numbers instead of fractions. Three attention checks were added to verify that participants were taking the time to compare the fractions.

Stimuli. For the fraction comparison task, a large set of stimuli was generated that contained all possible fraction comparisons using the numbers 1-32, along with a comparison classification based on its congruency, if they had common components, and their level of similarity to a musical context. No equivalent or improper fractions were used, and all fractions were in their most-reduced form. If a fraction pair could not be either congruent or incongruent (e.g., 7/16 vs 5/21), it was excluded. 172632 comparisons were generated. Two subsets of stimuli were generated from the initial set. The first subset contained 5 comparisons that were taken randomly from the initial comparison using the rand() in Microsoft Excel, and the numerators of both fractions were displayed as the whole numbers to be compared (e.g., original comparison was 3/7 vs 5/9, participants were shown 3 vs 5). This subset of stimuli was used for practice trials. The second subset contained 171 trials in total, the breakdown of which is summarized in Table 1 for incongruent comparisons and Table 2 for congruent comparisons. This subset was used for the real trials.

A comparison was said to be congruent if the numerator and denominator of the greater fraction were greater than or equal to those of the lesser fraction ($\frac{4}{5}$ vs $\frac{3}{5}$; $4 \geq 3$, $5 \geq 5$, $\therefore \frac{4}{5} \geq \frac{3}{5}$). Incongruent comparisons occurred when the numerator and denominator of the greater fraction were lesser than or equal to those of the lesser fraction (e.g., ½ vs ¼; $1 \leq 1$, $2 \leq 4$, $\therefore \frac{1}{2} \geq \frac{1}{4}$).

Fraction pairs were also classified on their transferability from a musical domain to a mathematical domain as either near transfer, moderately near transfer, moderately far transfer, or far transfer. A fraction comparison was near transfer if both fractions in the comparison had a numerator value of 1, and a denominator value of 2, 4, 8, 16, or 32. These values were chosen to mimic note lengths often encountered in music (e.g., ½ for half notes1/8 eighth notes). Moderately near transfer comparisons had denominators familiar to a musical context (i.e., 2, 4, 8, 16, or 32), and compared the same numerator, which could be any whole number except 1. Moderately far transfer comparisons also had denominators familiar to a musical context but compared different numerators, which could be any whole number except for 1. Moderately near and far transfer comparisons were implemented to determine the degree of change required for a comparison to no longer be considered familiar to a musical context. Assuming significant differences occurred between the comparisons that were familiar or unfamiliar to a musical context, if other levels of transfer comparison types insignificantly differed from one or the other, then they would be considered an extension of that context.

Fractions pairs with either the same numerator or denominator were classified as having common components; otherwise, they were classified as having no common components. Common components were classified to confirm with previous literature indicating an increase in the componential approach using common components (Ischebek et al., 2009; Meert et al., 2009; Obersteiner et al., 2013).

Practice Trials. Participants were given five practice trials; they were shown a fixation cross, followed by a set of two whole numbers. Whole numbers were used in the practice trials to reduce practice effects and strategy priming prior to the real trials. Participants were instructed to select the larger of the two numbers; they tapped the ‘F’ key or ‘J’ key to select the number on the left or right, respectively. ‘F’ and ‘J’ keys, aligned with the spatial layout of the fractions, and were utilized to reduce cognitive load, familiarizing participants with the spatial-kinesthetic method of selecting the larger magnitude using their keyboards. Data from practice trials were not included in the analysis.

Test Trials. For the real trials, participants performed a comparison task between two fractions; participants were shown a fixation cross, followed by a set of two fractions.

Participants were instructed to hit the 'F' key on their keyboard if they believed that the fraction on the left was larger, or the 'J' key if the fraction on the right was larger. Participants were given trials in blocks of 16, between which they had the option of indefinitely taking a break. Participants could end their break by pressing 'Spacebar'. To verify that participants were paying attention during the task, they were instructed to tap the 'P' key for trials 50, 93, and 150. The participants' keystrokes and response times were recorded for all real trials.

Arithmetic Task

For this task, participants evaluated a series of arithmetic expressions by typing in the solution into a textbox.

Stimuli. Using numbers from 0–9, we created every possible arithmetic expression with only two terms using addition, subtraction, multiplication, or division (e.g., $5 \div 1$). Afterwards, expressions without positive integer solutions were removed for a final total of 277 expressions.

Practice Trials. For the practice trials, participants were shown an arithmetic expression and were instructed to type in the correct answer using their keyboard. Participants could see their typed input in a text box. The practice trials lasted until participants had correctly evaluated five expressions. Feedback was provided after each expression (i.e., "Correct" or "Incorrect (Answer is __") to ensure participants understood the instructions before going into the real trials. Expressions were taken at random from the CSV file. Data from the practice trials was not included in the analysis.

Test Trials. The test trials were modelled after the Ben-Gurion University Math Fluency Test; this test was used because of its high concurrent validity for math fluency in a digital format (Gliksman et al., 2022). Participants were given three minutes to evaluate as many single-digit arithmetic expressions as they could. Like in the practice trials, they had to type in their response using their keyboard. Unlike the practice trials, they did not receive feedback between trials. All trials were randomized. Math fluency, operationalized as total number of correct answers, was used as a potential covariate in participants' response time and accuracy in the fraction comparison task. Performance on the arithmetic task was operationalized as number of correct answers to account for any speed-accuracy trade-offs that might have occurred.

Demographic Questionnaire

Participants were asked to complete a demographic questionnaire. The questionnaire contained questions asking participants for their strategies when completing each type of fraction comparison (e.g., comparing only numerators, using ½ as a reference point), and their perception of their own mathematical ability and music ability.

Data Analysis

All mean accuracies and response times were averaged per participant, per comparison type, and per participant group. Groups were compared against each other across each condition using multiple mixed ANOVAs. Statistically significant effects and interactions were further analyzed with Tukey-adjusted *t*-tests. All statistical analyses were conducted in R, and results with p-values of .05 or less were considered statistically significant.

Results

Fraction Comparison Task

Accuracy

Accuracy across all trials was high for both trained musicians (M = 87.4%) and non-musicians (M = 80.7%). A 2 (group) x 2 (congruency) x 2 (common components) mixed ANOVA revealed a significant main effect of common components, $F(1, 32) = 50.29$, $p < .001$. No other main effects were significant (see Fig. 1*A*). Participants answered comparisons that had common components more accurately (*M* = 0.89, *SE* = 0.03) than those without (*M* = 0.79, *SE* = .03), $t(32) = 7.09$, $p < .001$. A subsequent 2 (group) x 3 (common components) mixed ANOVA (2 x 3) was conducted to investigate the impact of different types of common components (common numerator, common denominator, and no common component) on response time. No new main effect or interaction was observed.

An additional 2 (group) x 4 (musical context) mixed ANOVA showed a main effect of musical context, $F(3, 96) = 13.69$, $p < .001$ (Fig. 1*B*). Moderately far transfer comparisons had the lowest accuracy of all (*M* = 0.78, *SE* = 0.03), followed by far transfer comparisons (*M* = 0.84, *SE* = 0.03), near transfer comparisons (*M* = 0.85, *SE* = 0.03), and moderately near transfer comparisons (*M* = 0.89, *SE* = 0.03). Moderately far transfer comparisons displayed significantly lower accuracies than far transfer comparisons [$t(32) = 3.50$, $p = .007$], near transfer comparisons [$t(32) = 4.39$, $p < .001$], and moderately near transfer comparisons [$t(32) = 5.77$, $p < .001$]. No other main effects or interactions were found.

A 2 (group) x 2 (congruency) x 2 (musical context) mixed ANOVA revealed a significant interaction of congruency and musical context, $F(1, 32) = 10.58$, $p = .003$ (Fig. 1*C*). This ANOVA was run with only near transfer and far transfer comparisons as factors under musical context due to a lack of congruent comparison trials for both moderately near and moderately far transfer comparisons. Participants had higher accuracy in incongruent fraction comparisons for near transfer comparisons (*M* = 0.90, *SE* = 0.03) as opposed to far transfer comparisons (*M* = 0.84, *SE* = 0.03), $t(32) = 5.15$, $p < .001$. There were no other significant main effects or interactions.

Response Time

A 2 (group) x 2 (congruency) x 2 (common components) mixed ANOVA revealed main effects of congruency [$F(1, 32) = 3.31$, $p < .001$] and common components [$F(1, 32) = 41.07$, $p < .001$], as well as interactions between group and congruency [$F(1, 32) = 14.29$, $p < .001$], and between group and common components [$F(1, 32) = 5.82$, $p = .022$; Fig. 2*A*]. No significant main effect of group was found, $F(1, 32) = 3.31$, $p = .078$. Participants generally responded faster in congruent trials ($M = 1.68$ s, $SE = 0.10$ s) compared to incongruent trials ($M = 1.85$ s, $SE = 0.12$ s), $t(32) = 4.81$, $p < .001$. Participants also responded faster when common components were present ($M = 1.44$ s, $SE = 0.08$ s) versus not present ($M = 2.09$ s, $SE = 0.15$ s), $t(32) = 6.41$, $p < .001$. Trained musicians responded faster to congruent comparisons ($M = 1.82$ s, $SE = 0.15$ s) than they did incongruent trials ($M = 2.11$ s, $SE = 0.18$ s), $t(32) = 5.74$, $p < .001$. Non-musicians did not significantly differ in their speed across congruency, $t(32) = 0.77$, $p = .446$. Non-musicians responded faster ($M = 1.77$ s, $SE = 0.20$ s) than musicians ($M = 2.41$ s, $SE = 0.22$ s) when there were no common components, $t(32) = 2.12$, $p = .042$.

To see if the type of common component (common numerator versus denominator vs no common component) affected response time, a follow-up 2 (group) x 3 (common components) mixed ANOVA was conducted (Fig. 2*B*). A main effect of common components was observed, $F(2, 64) = 37.36$, $p < .001$. A Tukey-adjusted pairwise comparison confirmed that having no common components resulted in slower response times ($M = 2.08$ s, $SE = 0.15$ s) than having a common numerator [$M = 1.51$ s, $SE = 0.09$ s; $t(32) = 6.41$, $p < .001$] or a common denominator [$M = 1.37$ s, $SE = 0.07$ s; $t(32) = 6.18$, $p < .001$], but also revealed that participants responded faster to trials with common denominators ($M = 1.37$ s, $SE = 0.07$ s) compared to common numerators [$M = 1.51$ s, $SE = 0.09$ s; $t(32) = 3.54$, $p = .004$]. Additionally, an interaction of group and common components was observed, $F(2, 64) = 6.01$, $p = .004$. A Tukey-adjusted *t*-test showed that trained musicians were fastest in comparisons with common denominators (*M* = 1.39 s, *SE* = 0.11 s) than with common numerators [*M* = 1.65 s, *SE* = 0.13 s; *t*(32) = 4.43, *p* < .001], while non-musicians were equally fast in comparisons with common denominators (*M* = 1.36 s, *SE* = 0.09 s) and common numerators [*M* = 1.38 s, *SE* = 0.11 s; *t*(32) = 0.34, *p* = 0.938]

A 2 (group) x 4 (musical context) mixed ANOVA highlighted showed a main effect of musical context, *F*(3, 96) = 26.35, *p* < .001. All levels of transfer significantly differed from each other (Fig 2*C*). Far transfer comparisons were the slowest (*M* = 2.06 s, *SE* = 0.14 s), followed by moderately far transfer comparisons (*M* = 1.83 s, *SE* = 0.13 s), then moderately near transfer comparisons (*M* = 1.57 s, *SE* = 0.10 s), and near transfer comparisons (*M* = 1.46 s, *SE* = 0.09). In addition, the interaction between group and musical context was significant, *F*(3, 96) = 3.07, *p* = .032. Trained musicians responded faster to near transfer comparisons (*M* = 1.55 s, *SE* = 0.14 s) than they did to moderately near transfer comparisons [*M* = 1.73, *SE* = 0.21; *t*(32) = 3.18, *p* = .016], or to moderately far transfer comparisons [*M* = 2.10 s, *SE* = 0.19 s; *t*(32) = 5.53, *p* < .001]. Non-musicians were equally fast for near transfer comparisons (*M* = 1.37 s, *SE* = 0.12 s), moderately near transfer comparisons [*M* = 1.42 s, *SE* = 0.13 s; *t*(32) = 1.01, *p* = .746] and moderately far transfer comparisons [*M* = 1.55 s, *SE* = 0.17 s; *t*(32) = 2.07, *p* = .184].

A 2 (group) x 2 (congruency) x 2 (musical context) mixed ANOVA showed an interaction of musical context and congruency, *F*(1, 32) = 23.99, *p* < .001 (Fig 2*D*). When faced with near transfer comparisons, participants responded faster for incongruent comparisons (*M* = 1.25 s, *SE* = 0.08 s) compared to congruent comparisons (*M* = 1.49 s, *SE* = 0.10 s), *t*(32) = 4.92, *p* < .001. Conversely, for far transfer comparisons, participants responded faster for congruent comparisons (*M* = 1.99 s, *SE* = 0.13 s), compared to incongruent comparisons (*M* = 2.16 s, *SE* = 0.15 s), *t*(32) = 3.07, *p* = .004. No significant interaction of group, musical context and congruency was seen, $F(1, 32) = 2.93$, $p = .097$. Aside from the main effect of near transfer and far transfer comparisons as per the previous 2 (group) x 4 (musical context) mixed ANOVA, no significant main effects were seen.

Balanced Integration Scores

Efficiency scores were calculated to determine if any observed effects resulted from an exacerbated speed-accuracy trade-off. The efficiency score calculation used was the balanced integration score (BIS) developed by Liesefeld and colleagues (2015). The BIS was chosen over other measures of combined performance (e.g., inverse efficiency score, rate-correct score, etc.) for its ability to retain the real effects while remaining insensitive to speed-accuracy trade-off at all accuracy levels (Liesefeld & Janczyk, 2019). BIS equally weighs response time and accuracy by standardizing both scores and subtracting the standardized response time score from the standardized accuracy score (Liesefeld & Janczyk, 2019). An average score is therefore 0; higher scores indicate stronger performance while lower scores indicate weaker performance (Liesefeld et al., 2015). The equation is seen below in (1), where *PC* represents the proportion of correct responses (accuracy), *RT* represents response time, and S_x represents the sample standard deviation of x:

$$BIS_{i,j} = z_{PC_{i,j}} - z_{RT_{i,j}},\ z_{x_{i,j}} = \frac{x_{i,j} - \bar{x}}{S_x} \tag{1}$$

The same analyses that were conducted for accuracy and response time were conducted for the BIS. A 2 (group) x 2 (congruency) x 2 (common components) mixed ANOVA only demonstrated a main effect of common components [$F(1, 32) = 103.29$, $p < .001$] and its interaction with group [$F(1, 32) = 9.25$, $p = .005$; Fig 3*A*].

Additionally, a 2 (group) x 2 (congruency) x 2 (musical context) mixed ANOVA had a different interaction of musical context and congruency (Fig 2*B*). For near transfer comparisons, participants had significantly stronger performance in the incongruent comparisons (M = 0.84, SE= 0.17) compared to congruent comparisons (M = 0.25, SE = 0.13), $t(32) = 4.58$, $p < .001$. However, participants' performance for far transfer comparisons were no longer significantly different between incongruent (M = -0.43, SE = 0.17) and congruent comparisons (M = -0.24, SE = 0.16), $t(32) = 1.34$, $p = .190$.

The 2 (group) x 4 (musical context) mixed ANOVA did find a main effect of musical context, $F(3, 96) = 30.03$, $p < .004$ (Fig 3*C*). Post-hoc *t*- tests yielded results consistent with those of accuracy for the same conditions. A 2 (group) x 2 (congruency) x 2 (musical context) mixed ANOVA had similar results to the same analysis on response time.

Demographic Questionnaire

In incongruent comparisons with common denominators, the most popular strategy used by participants was comparing denominators, both in near transfer (M = 49.2%, n = 65) and far transfer comparisons (M = 43.8%, n = 64; Fig 4*A*, *B*). Similarly, congruent comparisons with common numerators were associated with comparing only numerators in near transfer comparisons (M = 54.7%, n = 64) or far transfer comparisons (M = 73.8%, n = 65; Fig 4*C*, *D*).

For near transfer, incongruent comparisons without common components, 46.2% of participants said they would convert the fractions into like fractions (fractions with the same denominator), while 44.6% report that they would compare the numerators and the denominators together (n = 65; Fig 4*E*). In far transfer, incongruent comparisons without common components, the majority of participants compared numerators and denominators together (M = 35.4%, n = 65), or used benchmarks like ½ as reference points to compare fractions (M = 33.8%, n = 65; Fig 4*F*).

For near transfer, congruent comparisons without common components, most participants reported comparing numerators and denominators (*M* = 47.7%, *n* = 65), or converting the fractions in comparisons into like fractions (*M* = 46.2%, *n* = 65; Fig 4*G*). In far transfer, congruent comparisons without common components, participants compared numerators and denominators (*M* = 47.7%, *n* = 65), or examined the gap between the numerator and the denominator (*M* = 38.5%, *n* = 65; Fig 4*H*).

Discussion

This study aimed to investigate differences in fraction comparison performance between musicians and non-musicians, as well as to investigate potential moderators of the whole number bias between the two groups. Specifically, we examined how congruency, common components, and similarity of comparisons to a musical domain affected accuracy and response time when participants compared fractions. The results revealed several noteworthy findings, which have been summarized below.

Musicians vs Non-musicians

Generally, trained musicians and nonmusicians had equal response times and accuracies within significance. However, significant differences uniquely observed within the trained musicians group highlight potential disparities between the participant groups. Trained musicians responded faster to near transfer comparisons than they did to moderately near transfer comparisons. Moreover, no difference was found between near transfer and moderately near transfer comparisons in non-musicians, suggesting that this improvement can be attributed to musical training. This finding is in line with our prediction, as near transfer comparisons are directly related to note lengths, whereas the moderately near transfer comparisons are not specific to a musical domain. Musicians had no advantage over non-musicians for moderately near transfer comparisons. While musicians showed distinct response patterns between near transfer and moderately near transfer comparisons, non-musicians exhibited no such differentiation, indicating a lack of transfer effects in their fraction comparison abilities. This highlights possible cognitive advantages associated with musical expertise.

Moderators of the Whole Number Bias

Congruency

Firstly, participants exhibited high accuracy rates regardless of congruency, with trained musicians generally but insignificantly outperforming non-musicians. The persistently high accuracy across conditions and groups is consistent with previous literature on the whole number bias in adult populations (Obersteiner et al., 2013, 2016; Vamvakoussi et al., 2012). The sample for the study consisted of 18- to 23- year-old university students; it has been noted that adults do not often make mistakes when comparing fractions (Obersteiner et al., 2013; Vamvakoussi et al., 2012). Instead, the whole number bias manifests in adult populations as slower response times for incongruent comparisons compared to congruent comparisons (Meert et al., 2010a).

Response times were faster on average when responding to congruent trials, which is coherent with the dual-processing theory suggested by Gillard et al. (2009). Dual-processing theory suggests that participants first approach the comparison by addressing the fractions' components separately but then suppress the componential method of comparing fractions for a more holistic approach when facing incongruent comparisons (DeWolf & Vosniadou, 2015; Vamvakoussi et al., 2012). This inhibition of the componential approach takes time, which results in slower response times from participants (Vamvakoussi et al., 2012)

Common Components

Regardless of congruency, participants had faster response times when comparing fractions with common components. The faster speeds indicate a componential approach for these types of comparisons. This result is consistent with the previous literature (Meert et al., 2010b; Vamvakoussi et al., 2012), as having common denominators automatically makes for a congruent comparison, and common numerators create incongruent comparisons. These findings align with previous research suggesting that individuals may rely on heuristic strategies based on common components to simplify complex decision-making processes. Obersteiner and colleagues (2013) discuss how common components can exacerbate effects of the whole number bias, even among expert mathematicians. Meert and colleagues (2010a) also suggest that fraction processing could exist on a continuum of holistic to componential approaches, rather than one or the other. Under that framework, common components would mark the extreme on the componential approach, and while congruent fractions without common components may still merit componential approaches, it may also result in students accessing the magnitude itself or using other comparison methods. This continuum framework for fraction processing is supported by the demographics questionnaire administered to participants in the present study. When faced with common components, most of the participants reported comparing only the uncommon component as a heuristic. However, for congruent comparisons without common components, substantially-less participants reported using this strategy, and other strategies emerged, such as converting the fraction pair to like fractions. There is an even more diverse selection of strategies for incongruent comparisons without common components, including using other magnitudes as benchmarks from which to relatively estimate. The benchmark strategy requires participants to consider not just the components separately, but the approximate magnitudes of the fractions to be compared before judging a fraction to be greater than the other. Overall, these findings support that response time in fraction comparisons is influenced by decision-making processes and other cognitive processes.

Music Context

The observed effects of musical context on accuracy, response time, and balanced integration scores underscore the potential effects of musical training on mathematical tasks. All participants displayed higher accuracy and faster response times when comparisons were near transfer. The homogeneity in results across participant groups suggests that factors beyond familiarity with musical notation facilitated performance in these comparisons. A closer examination of not just the musical context, but also the numerical structure of the comparisons provides a possible explanation. Near transfer comparisons contain fractions that share a common numerator of 1. The present study and previous literature have found common numerators to be associated with faster response times and higher accuracies (Meert et al., 2010a). This association possibly confounds the results of the near transfer comparisons; it is not possible in the current paradigm to verify if the faster response times from those trials are due to the structure of the fractions in the pair, or the transferability of the comparison from a musical to a mathematical domain.

Implications

The findings of this study add to the literature in the field of numerical cognition and lay the ground for related future causal studies. These results have important implications for education and practice. It is important to note that the increased response time due to the whole number bias is not necessarily negative, and that faster response times are not the overarching educational goal. Rather, the goal is to reduce erroneous judgement influenced by the whole number bias. Although response times may provide insight as to the cognitive load required to respond to the comparisons, ultimately, our objective is for students to understand proportions in a manner that makes learning rational numbers more intuitive. Implementing other strategies may lead to shorter response times in adults who are already proficient in fraction comparison. However, younger students may experience fewer errors and gain a more accurate conceptual understanding of fractions through alternative teaching methods. Educators should therefore not be focused on students' speed, but on their ability to distinguish between whole numbers and rational numbers.

Limitations

Despite the valuable insights gained from this study, several limitations must be acknowledged. The study relies on measures of self-report to demographic questions to classify participants into different participant groups. However, it is possible that some participants classified as non-musicians may have had sufficient music experience to perform to the same standards as musicians on all comparison tasks. Integrating a metric of musicianship or a task that does not rely on self-report into the study design and using results from that metric to, in part, assign participants to groups would add to the criterion validity of the study.

Furthermore, the task design may not fully capture the complexity of cognitive processes involved in fraction comparison. In addition to congruency and common components, there may be other factors that influence comparison ability, such as the degree of musical training or mathematical ability. Additionally, individual differences in musical training and mathematical ability were not fully accounted for in the analysis, which may have influenced the results.

Musicians answered significantly more questions correctly on the arithmetic task, indicating a greater math fluency in the population. Although it may not be the case that math fluency predicts performance in fraction comparison tasks, in the current study design it is not possible to ascertain if it is indeed a predictor. Therefore, it would be presumptuous to attribute greater performance of musicians over nonmusicians to a transfer of skill from the domain of music to mathematics, instead of a general mathematical skill. It may be the case that music ability in general improves comparison ability. Future experiments could incorporate self-taught musicians who cannot read sheet music as a third experimental group to discriminate between effects of music ability in general versus music training.

An additional issue with self-report is that it relies on self-perceived strategy, and therefore does not shed light on the underlying, subconscious cognitive mechanisms behind the inhibition of an unusable strategy. While response time provides some feedback as to when participants are making decisions, it cannot tell you what exactly is happening, only that there likely is something occurring. Incorporating neuroimaging technology or electrophysiology in future studies may help to clarify what is occurring on a biological level of analysis, helping to quantify the inhibition process.

Although individuals may not be musicians, it does not preclude them from having external exposure to fractions used within a musical context, albeit in a different setting. Other professionals and hobbyists such as chefs or artists may refer to binary divisions regularly within their work in a non-numerical way. If this is the case, then they may exhibit similar behaviour as musicians when comparing fractions deemed as being present within a musical context. Although participants were inquired for other professions in which they may have had experience, they were not classified as another participant group. Further studies should investigate the effects of the whole number bias in other professions with fraction expertise.

Conclusion

In conclusion, this study provides novel insights into the relationship between musical expertise, cognitive processes, and numerical cognition. By investigating the impact of musical context, congruency, and common components on fraction comparison performance in musicians, this study contributes to our understanding of how cognitive abilities are influenced by domain-specific factors. Future research should further explore the mechanisms underlying these effects and investigate the long-term cognitive benefits of music training on mathematical abilities.

Acknowledgements

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Appendix A

Table A1

Summary of Trial Breakdown in Incongruent Comparisons

Musical Context	Common Components	No Common Components
Far transfer	14	14
Near transfer	20	0
Moderately near transfer	33	0
Moderately far transfer	0	20

Note. The table provides a breakdown of trial conditions in incongruent comparisons categorized by musical context and the presence of common components. ‘Far transfer’ denotes comparisons unrelated to musical notation, while ‘Near transfer" refers to comparisons directly related to note lengths. ‘Moderately near transfer’ and ‘Moderately far transfer’ represent comparisons with varying degrees of association to musical notation. Moderately near transfer comparisons contained fraction pairs with the same numerator, and denominators that were powers of 2. Moderately far transfer comparisons contained fraction pairs without the same numerator, and denominators that were powers of 2.

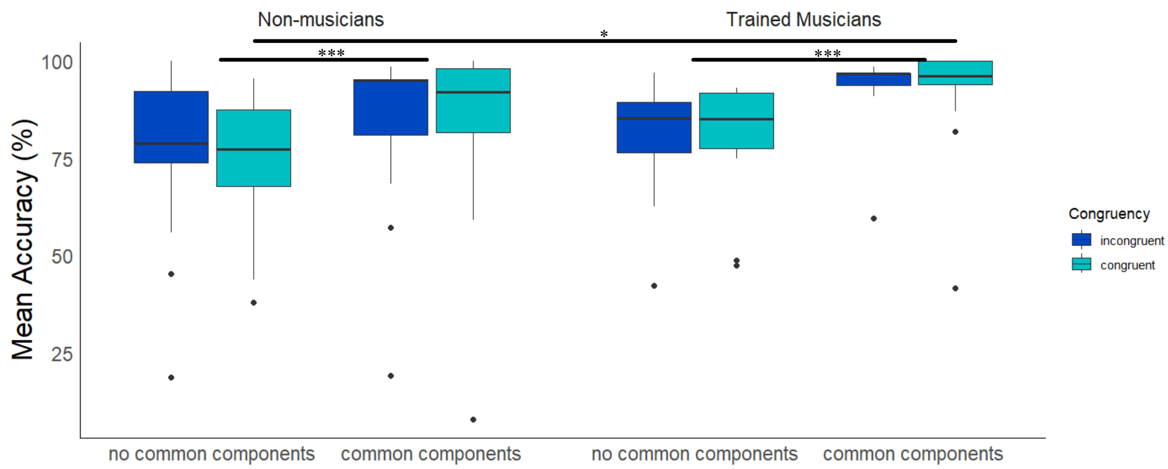
Table A2

Summary of Trial Breakdown in Congruent Comparisons

Musical Context	Common Components	No Common Components
Far transfer	14	14
Near transfer	8	0
Moderately near transfer	0	0
Moderately far transfer	3	11

Note. The table provides a breakdown of trial conditions in congruent comparisons categorized by musical context and the presence of common components. 'Far transfer' denotes comparisons unrelated to musical notation, while 'Near transfer" refers to comparisons directly related to note lengths. 'Moderately near transfer' and 'Moderately far transfer' represent comparisons with varying degrees of association to musical notation. Moderately near transfer comparisons contained fraction pairs with the same numerator, and denominators that were powers of 2. Moderately far transfer comparisons contained fraction pairs without the same numerator, and denominators that were powers of 2.

Appendix B



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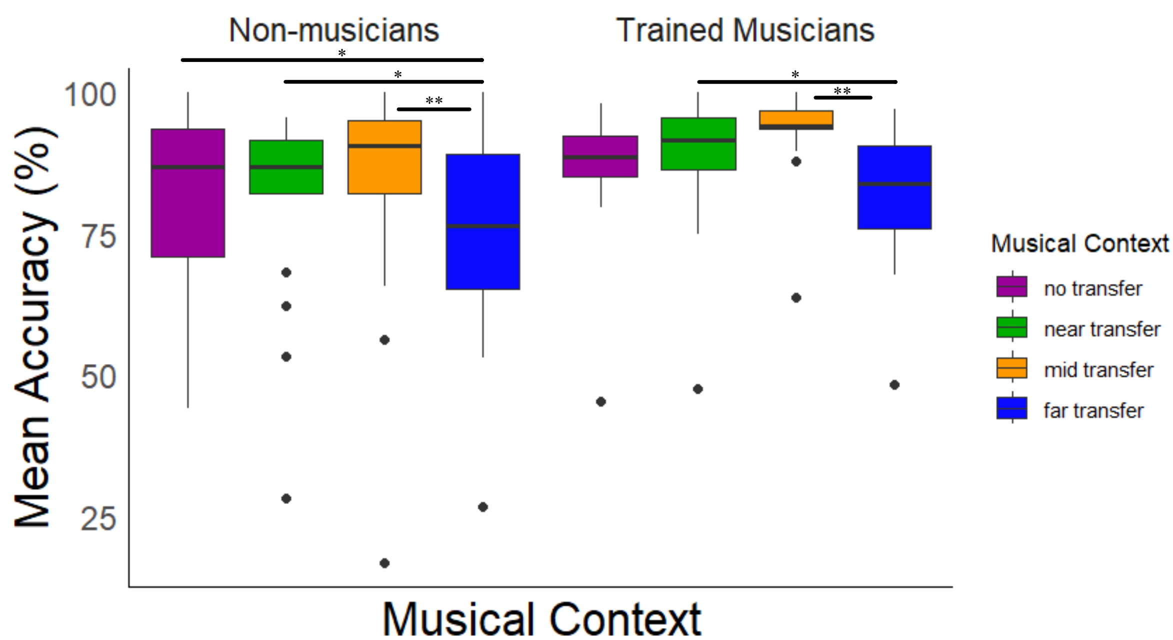

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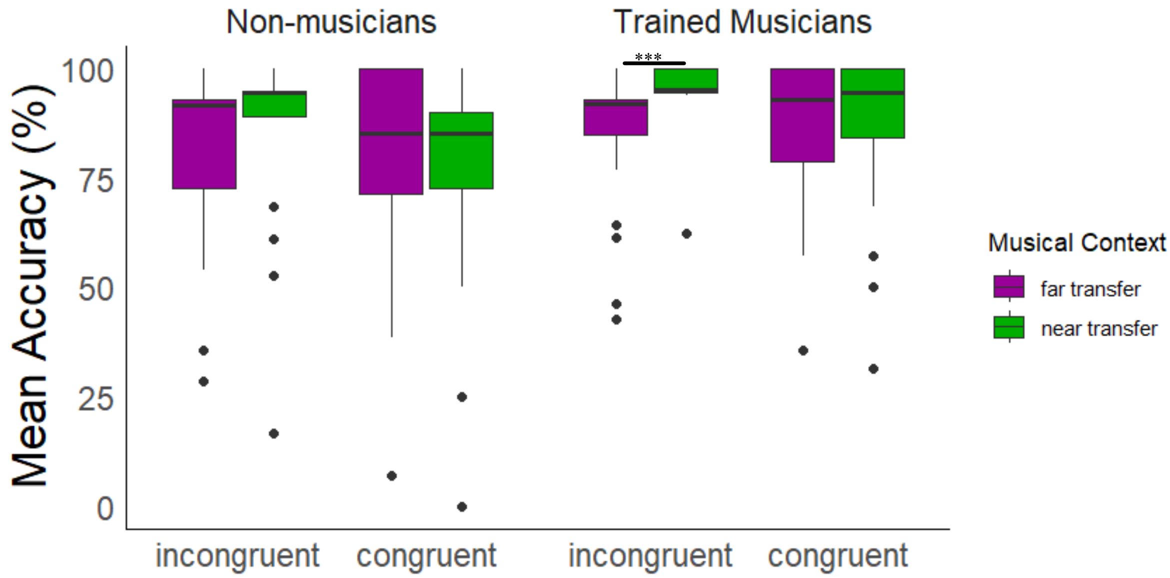
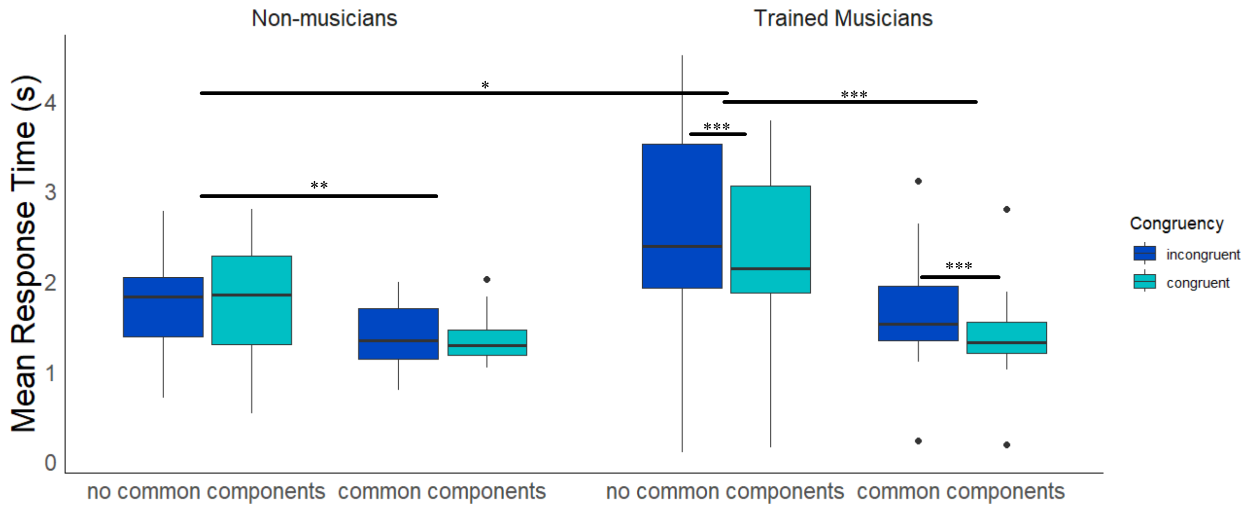


C.

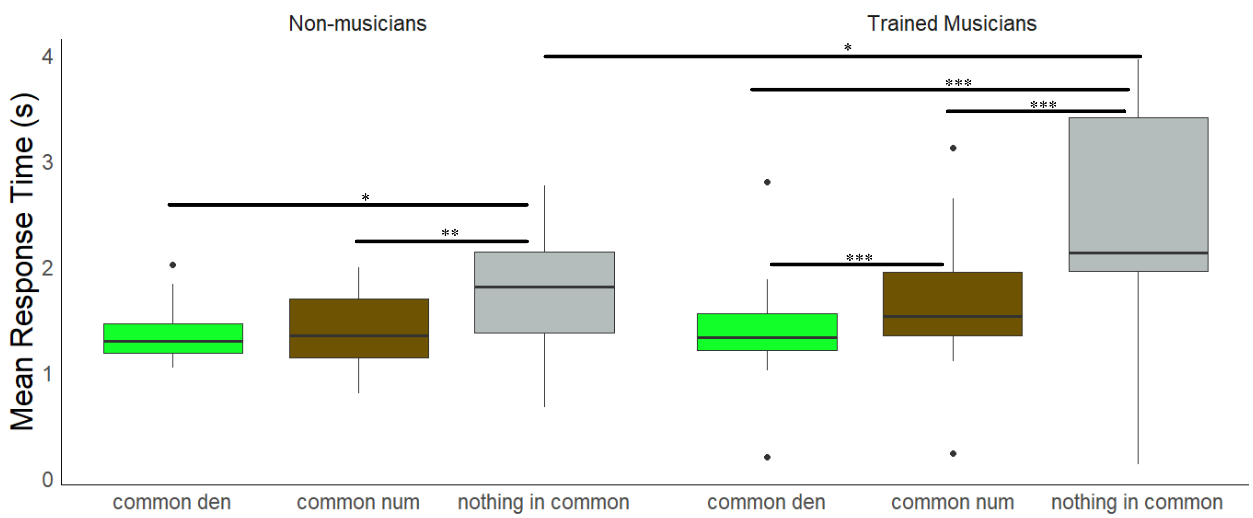
Fig. 1. Box plots representing accuracy scores across different conditions in fraction comparison tasks. Each box plot corresponds to a specific analysis conducted using mixed ANOVA. (*A*) Accuracy across all trials for both trained musicians and non-musicians, indicating a significant main effect of common components ($p < .001$). (*B*) Main effect of musical context, showing that moderately far transfer comparisons resulted in the lowest accuracy scores ($p < .001$). (*C*) Interaction between congruency and musical context, highlighting higher accuracy in incongruent fraction comparisons for near transfer comparisons compared to far transfer comparisons ($p < .001$). The box plot elements include the median (line inside the box), interquartile range (IQR, box), range up to 1.5 times the IQR (whiskers), and outliers (individual data points beyond 1.5 times the IQR).

*$p < .05$, ** $p < .01$, *** $p < .001$

A.


Non-musicians
Trained Musicians
Mean Response Time (s)
0
1
2
3
4
*
***
***
**
***
Congruency
incongruent
congruent
no common components
common components
no common components
common components

B.


Non-musicians
Trained Musicians
Mean Response Time (s)
0
1
2
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**
***
common den
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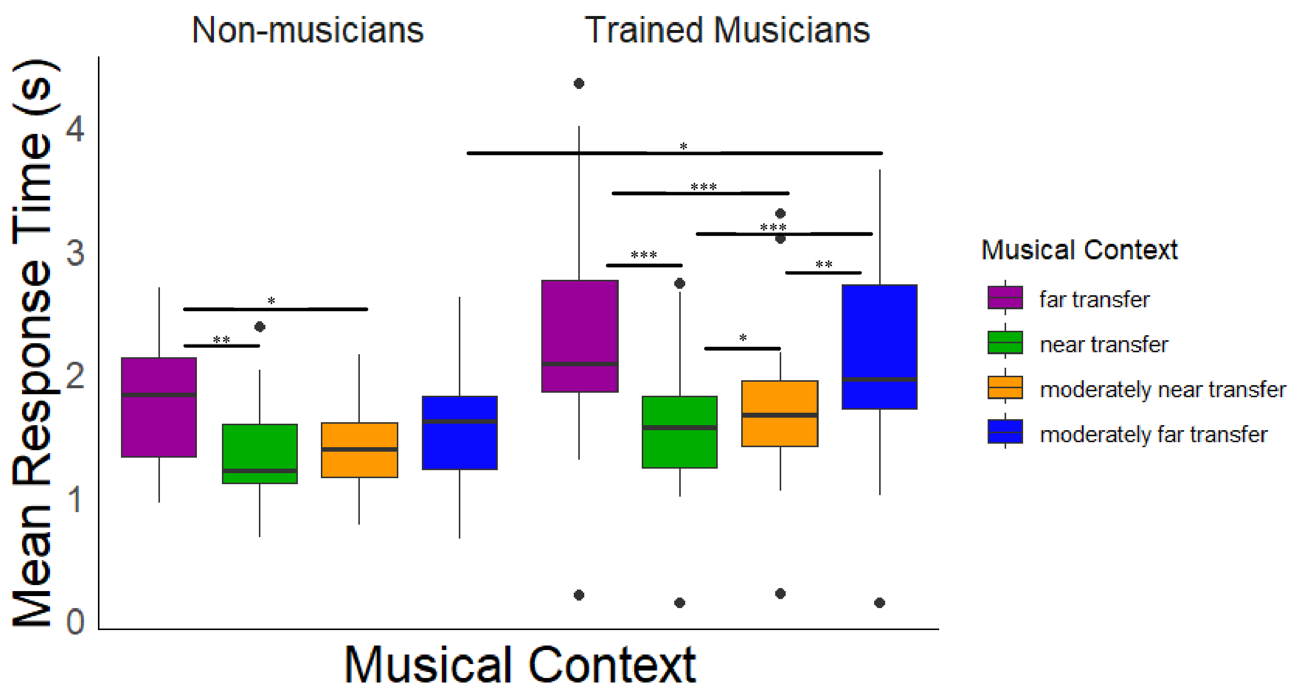
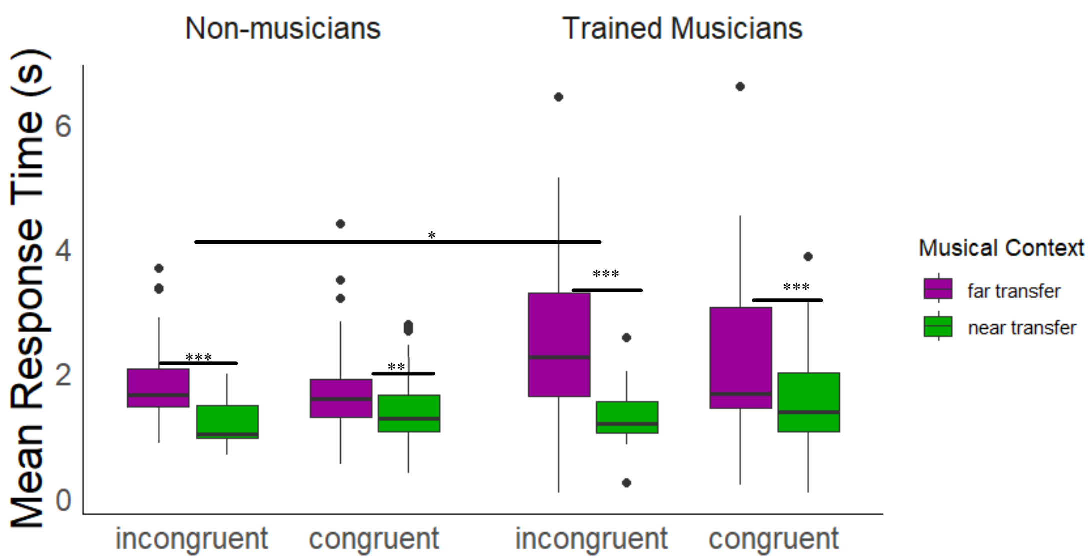
nothing in common
common den
common num
nothing in common

C.





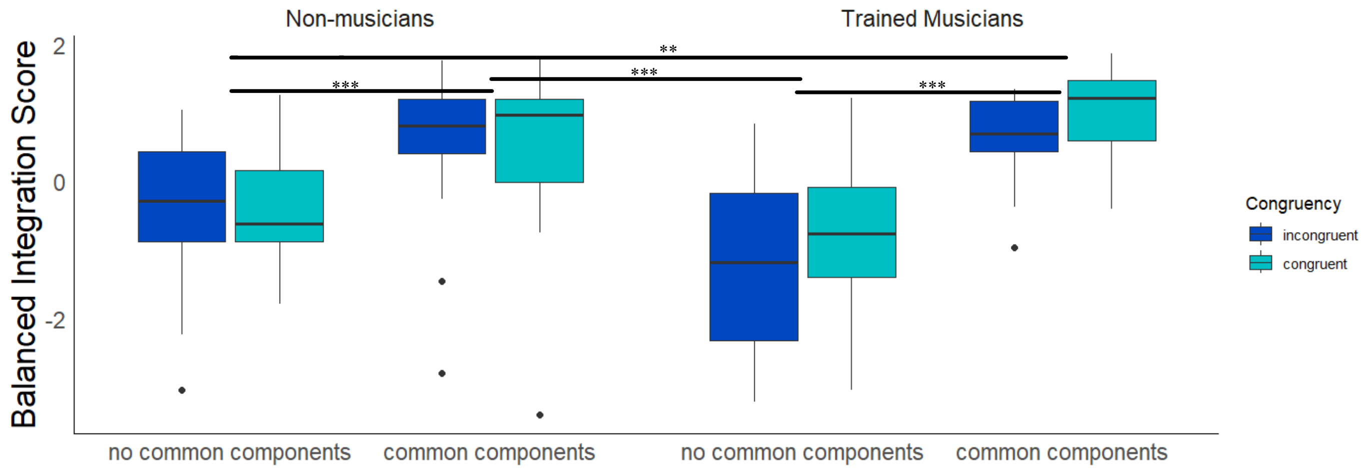
D.

Fig. 2. Box plots illustrating response times across different conditions in fraction comparison tasks. Each box plot corresponds to a specific analysis conducted using mixed ANOVA. (*A*) Main effects and interactions of congruency and common components on response times, indicating faster responses in congruent trials and when common components are present ($p < .001$). (*B*) Follow-up analysis investigating the effect of common components on response

time, showing that trials with common denominators elicited faster responses compared to trials with common numerators ($p < .001$). (*C*) Main effect of musical context on response times, with significant differences between all levels of transfer. (*D*) Interaction between group and musical context, indicating that trained musicians responded faster to near transfer comparisons compared to mid and moderately far transfer comparisons ($p < .001$). Additional interaction of musical context and congruency, revealing faster responses in incongruent comparisons for near transfer comparisons and in congruent comparisons for far transfer comparisons ($p < .001$).

*$p < .05$, ** $p < .01$, *** $p < .001$

A.


Non-musicians
Trained Musicians
Balanced Integration Score
2
0
-2
**
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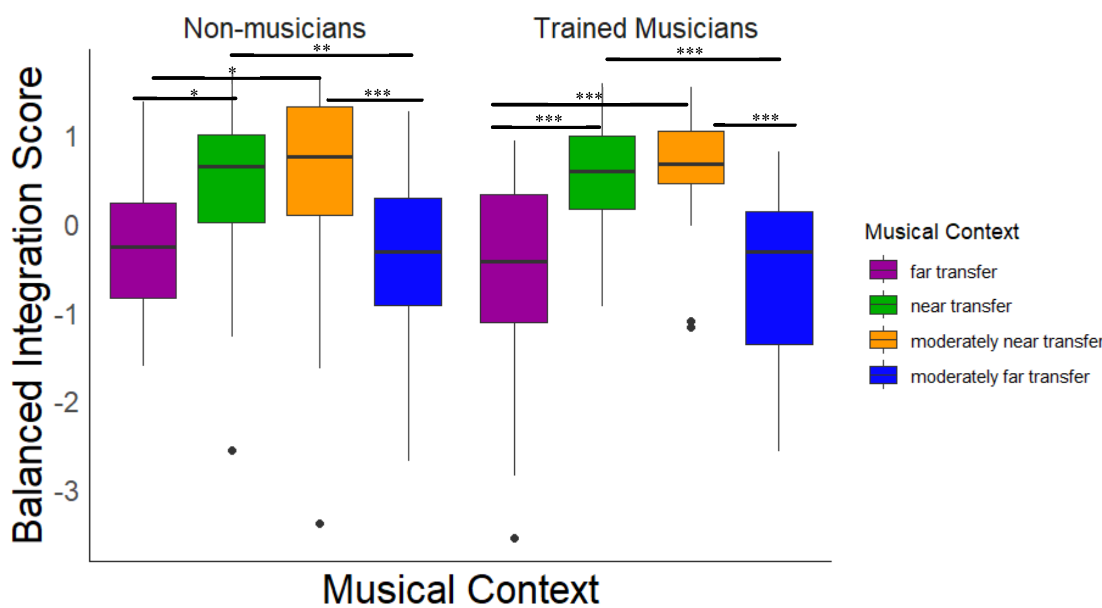
***
***
no common components
common components
no common components
common components
Congruency
incongruent
congruent


Non-musicians
Trained Musicians
Balanced Integration Score
1
0
-1
-2
-3
*
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Musical Context
Musical Context
far transfer
near transfer
moderately near transfer
moderately far transfer

B.

C.

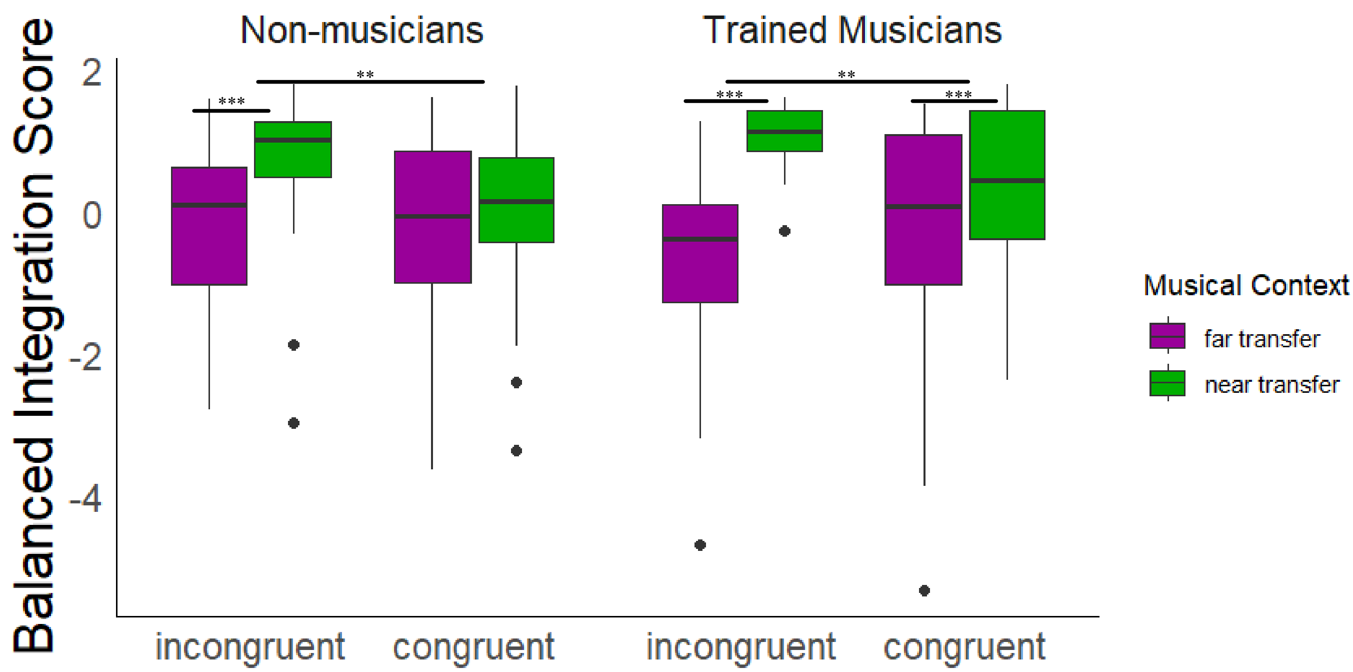
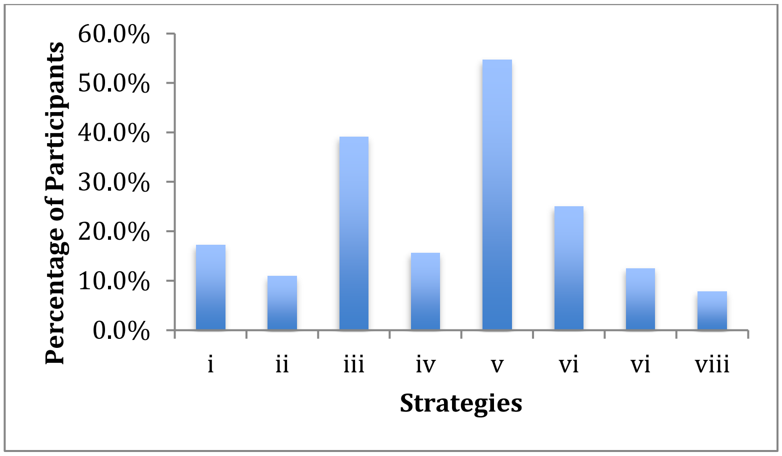


Fig. 3. Box plots illustrating the Balanced Integration Scores (BIS) across different conditions in fraction comparison tasks. The BIS combines response time and accuracy into a single measure of performance, calculated using the equation: BIS = $z_{PC} - z_{RT}$, where PC represents the proportion of correct responses (accuracy), RT represents response time, and z_x represents the standardization of x scores. Higher BIS scores indicate stronger performance, while lower scores indicate weaker performance. (*A*) Main effects and interactions of congruency and common components on BIS, revealing a significant main effect of common components and its interaction with group ($p < .001$). (*B*) Interaction of musical context and congruency on BIS, showing significant differences between incongruent and congruent comparisons for near transfer conditions ($p < .001$). (*C*) Main effect of musical context on BIS, with consistent patterns observed as in accuracy analyses.

*$p < .05$, ** $p < .01$, *** $p < .001$

Appendix C



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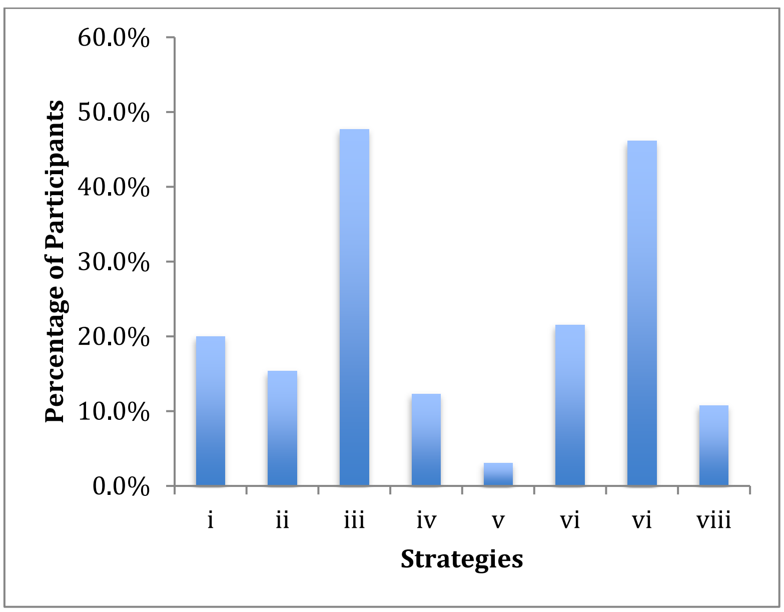
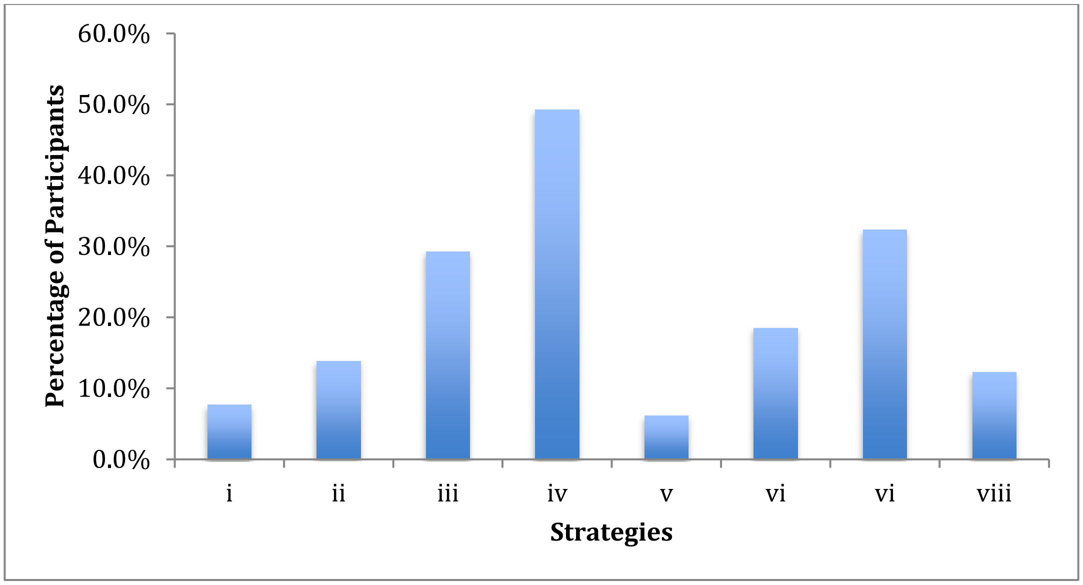


B.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Percentage of Participants
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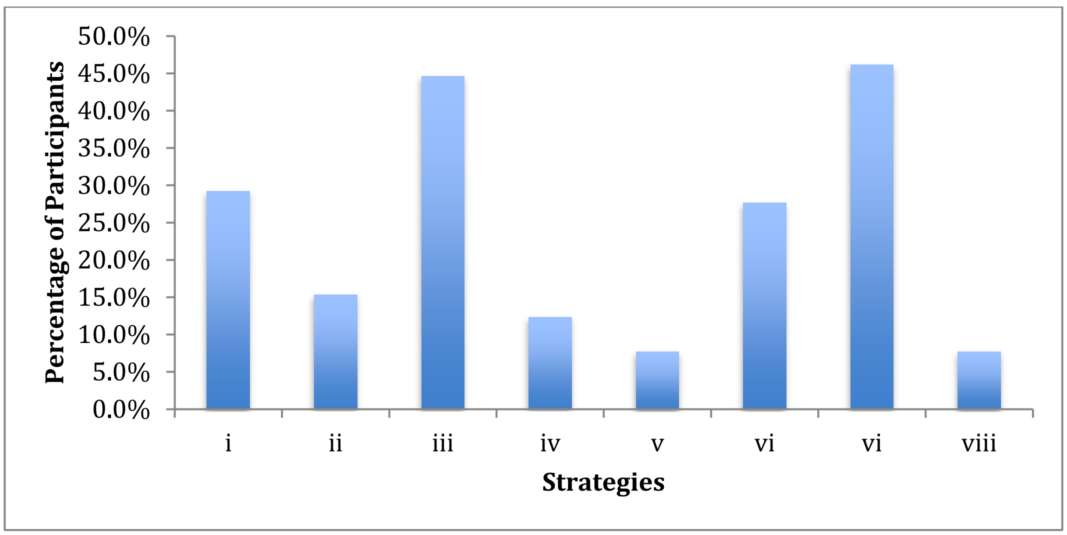
viii
Strategies

C.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Strategies

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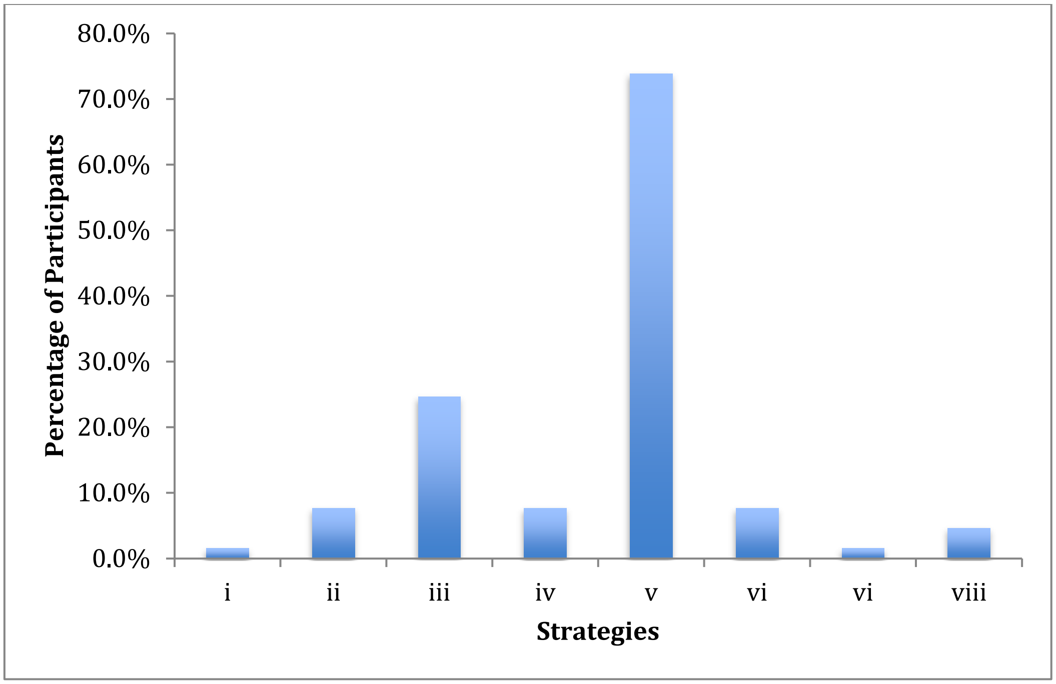
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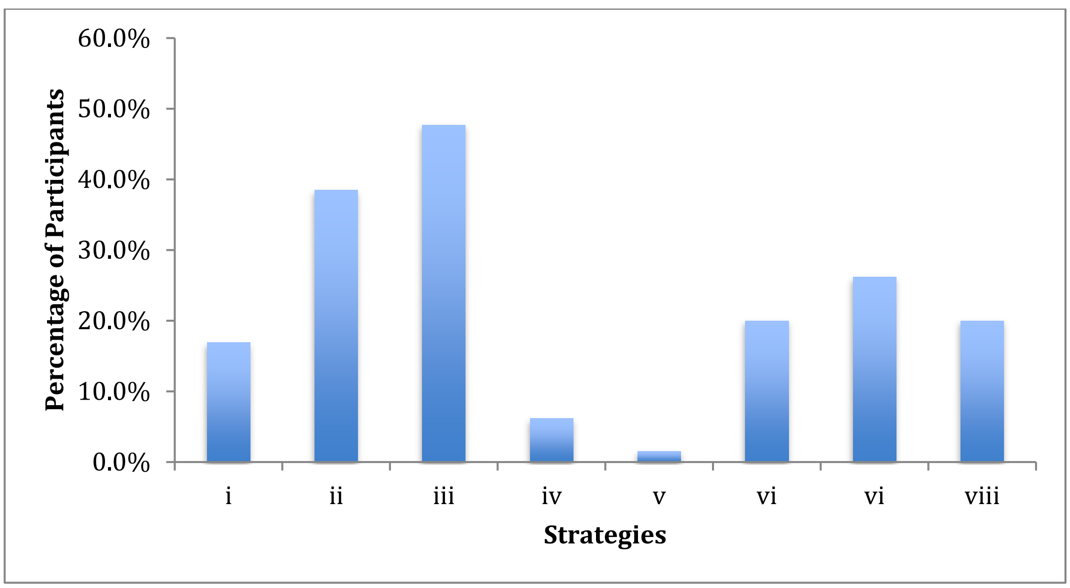
Strategies

E.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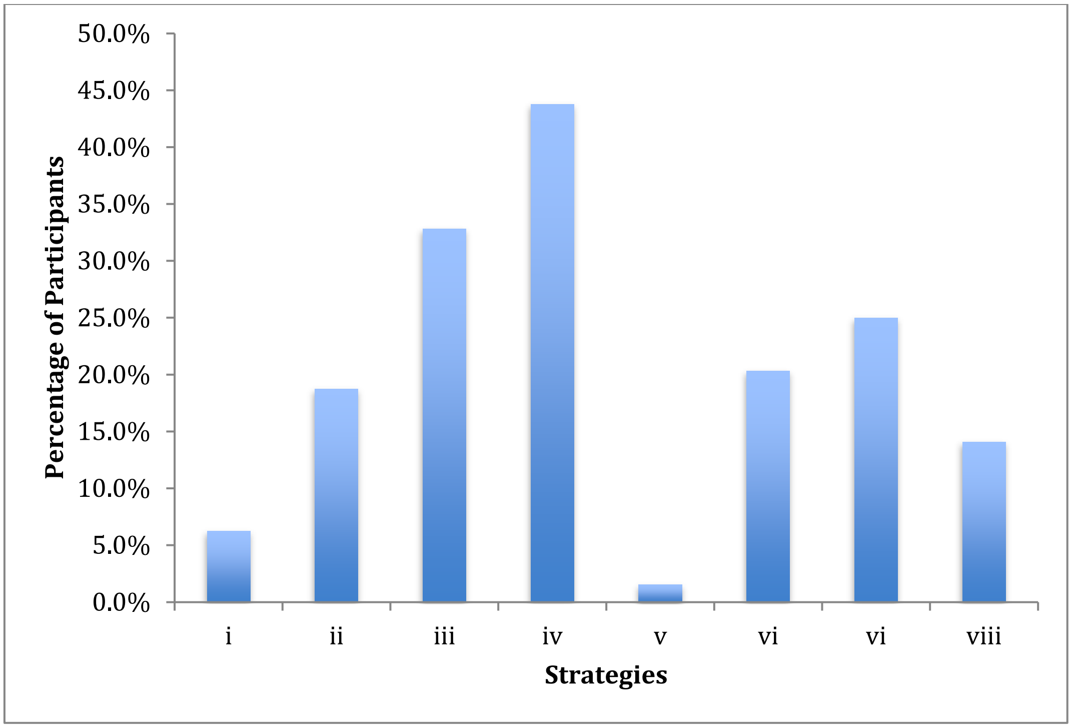
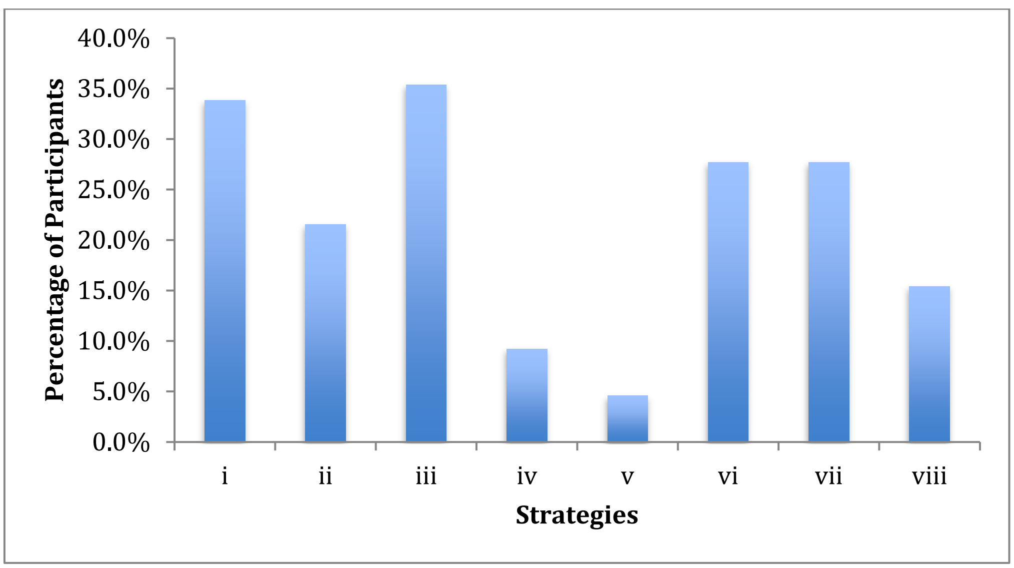
Strategies

F.



G.



H.

Fig. 4. Bar graphs illustrating strategies used for each type of fraction comparison. Roman numerals represent the strategies used: i = Benchmarks for reference points (e.g., 3/10 < 1/2 < 8/10); ii = Comparing gap between numerator and denominator in each fraction; iii = Comparing numerators and denominators together; iv = Comparing only denominators; v = Comparing only numerators; vi = Converting fractions into decimals/percentages; vii =

Converting into like fractions (same denominators); viii = Cross Multiplication (multiplying the numerator/denominator of one fraction with the denominator/numerator of the second). (*A*) The most popular strategy used in near transfer, incongruent comparisons with common components was to compare the denominators [$M = 49.2\%$, $n = 65$]. (*B*) For far transfer comparisons with common numerators, 43.8% of participants compared the denominators ($n = 64$). (*C*) 54.7% of participants compare just the numerators when faced with congruent, near transfer comparisons with common components ($n = 64$). (*D*) In far transfer comparisons with common denominators, participants commonly compare the numerators [$M = 73.8\%$, $n = 65$]. (*E*) In near transfer, incongruent comparisons without common components, 46.2% of participants report converting the fractions into like fractions, while 44.6% report comparing the numerators and the denominators together ($n = 65$). (*F*) The majority of participants compared numerators and denominators together [$M = 35.4\%$, $n = 65$], or used benchmarks like ½ as reference points to compare fractions [$M = 33.8\%$, $n = 65$], when faced with far transfer, incongruent comparisons without common components. (*G*) Participants reported comparing numerators and denominators [$M = 47.7\%$, $n = 65$], or converting the fractions in comparisons into like fractions [$M = 46.2\%$, $n = 65$] for near transfer, congruent comparisons without common components. (*H*) Participants either compared numerators and denominators together [$M = 47.7\%$, $n = 65$], or examined the gap between the numerator and the denominator [$M = 38.5\%$, $n = 65$] when examining far transfer, congruent comparisons without common components.